EE 232 Lightwave Devices
Lecture 4: Density of States, Quantum Wells and Wires

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Review of Quantum Mechanics

Schrodinger Equation

$$H\Psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t)$$

$$H = \frac{P^2}{2m} + V(\vec{r},t)$$ Hamiltonian = Kinetic + Potential Energy

$\Psi(\vec{r},t)$ Wavefunction

$\left|\Psi(\vec{r},t)\right|^2 = \Psi(\vec{r},t)\cdot\Psi(\vec{r},t)^*$ Probability of finding particle at $\vec{r}$

$\vec{P} = -i\hbar\nabla$ Momentum operator

$\left\langle\vec{P}\right\rangle = -i\hbar\int\Psi(\vec{r},t)^*\nabla\Psi(\vec{r},t)d\vec{r}$ Average Momentum

$\left\langle\vec{r}\right\rangle = \int\Psi(\vec{r},t)^*\vec{r}\Psi(\vec{r},t)d\vec{r}$ Average Position

Electron Plane Wave

$$\Psi(\vec{r},t) = e^{i\vec{k}\cdot\vec{r} - i\omega t}$$

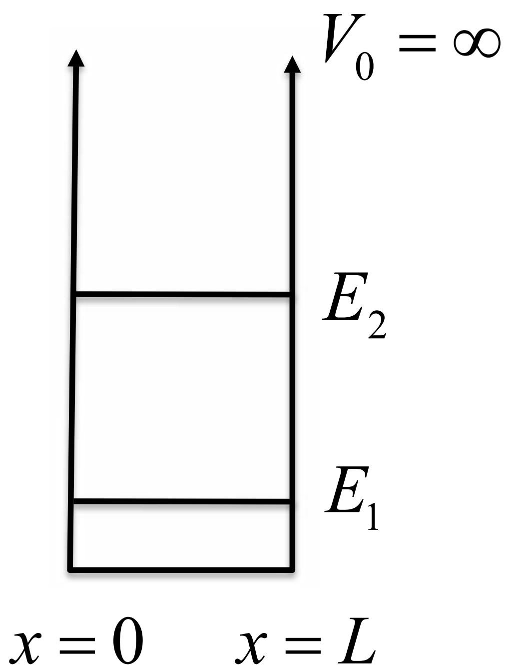
$$\text{LHS:} \quad H\Psi(\vec{r},t) = \frac{\hbar^2 k^2}{2m}\Psi(\vec{r},t) + V(\vec{r},t)\Psi(\vec{r},t)$$

$$P = \hbar k$$

$$\text{RHS:} \quad i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = \hbar\omega\Psi(\vec{r},t)$$

$$\Rightarrow \frac{\hbar^2 k^2}{2m} + V(\vec{r},t) = \hbar\omega$$

Example: Infinite Potential Well



Time Independent Potential

$$V(z) = \begin{cases} 0 & \text{for } 0 < z < L \\ \infty & \text{for } z < 0 \text{ or } z > L \end{cases}$$

$$\Psi(z,t) = \phi(z) e^{-i\omega t}$$

Solve Eigenvalue $E = \hbar\omega$ in

$$-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}\phi(z) + V(z)\phi(z) = E\phi(z)$$

For $0 < z < L$, $\quad \dfrac{d^2}{dz^2}\phi(z) + \dfrac{2mE}{\hbar^2}\phi(z) = 0$

$$\phi(z) = \begin{cases} \sin(kz) \\ \cos(kz) \end{cases}$$

B.C. $\quad \phi(z = 0) = \phi(z = L) = 0$

$$\phi_n(z) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} z\right)$$

$$E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2$$

Typical Examples

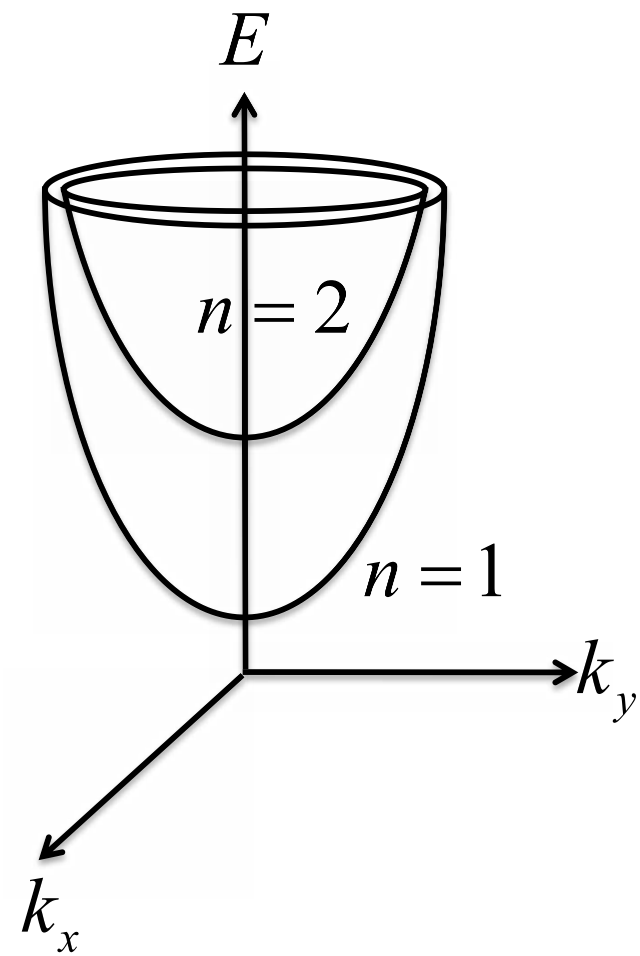
In GaAs, $m_e^* = 0.067 m_0$

For a 10-nm-wide potential well $(L = 10nm)$

$E_1 = 56$ meV

$E_2 = 4E_1 = 224$ meV

Complete Wavefunction for Infinite Potential Well



$$\Psi(\vec{r},t) = \phi'(x,y)\phi(z)e^{-i\omega t}$$

Electron confined in z, but free in x, y

$\Rightarrow$ Plane wave in x and y

$$\phi'(x,y) = \frac{1}{\sqrt{A}} e^{ik_x x + ik_y y}$$

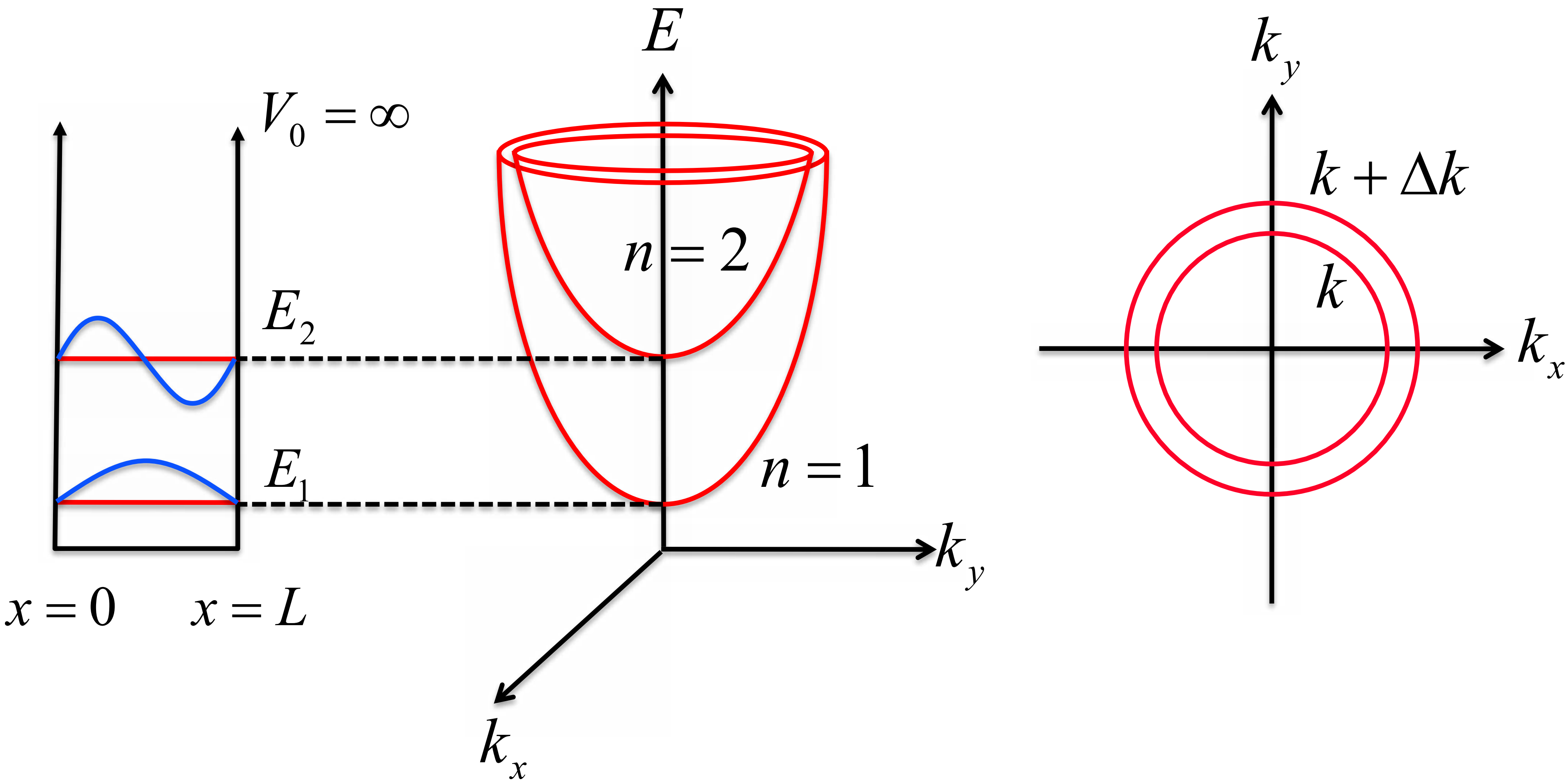
A: area (normalization const)

$$\Psi(\vec{r},t) = \sqrt{\frac{2}{L}} \frac{1}{\sqrt{A}} e^{ik_x x + ik_y y} \sin\left(\frac{n\pi}{L} z\right)$$

$$E_n = \frac{\hbar^2}{2m}\left[k_x^2 + k_y^2 + \left(\frac{n\pi}{L}\right)^2\right]$$

Energy quantized only in k_z direction

2-d Density of States



2-d Density of States

Consider the lowest band first (n=1):

Number of electron states between k and $k + \Delta k$ per unit volume

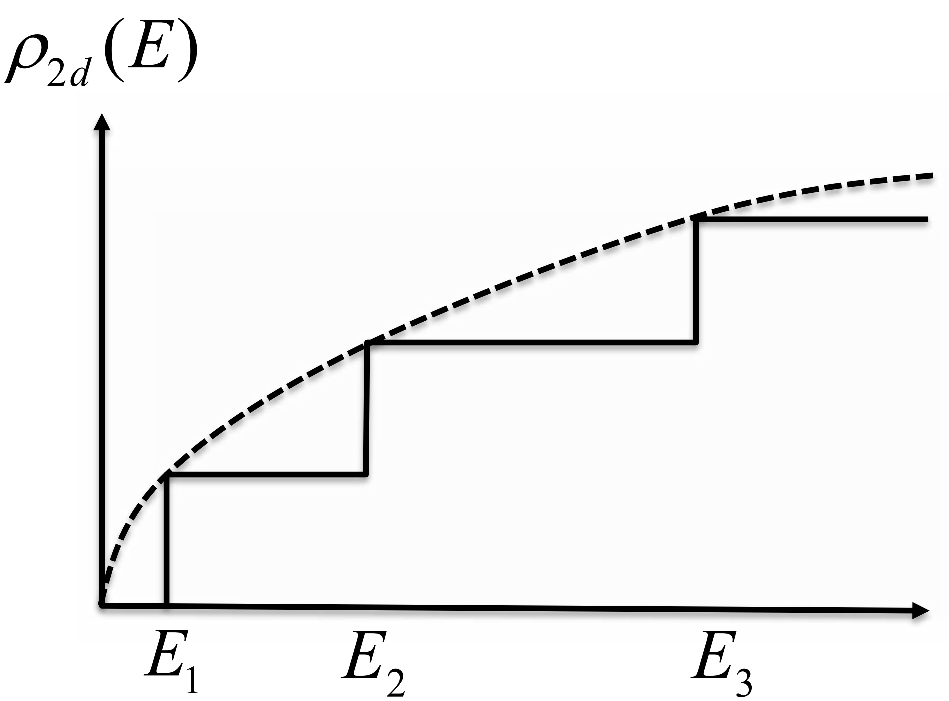
$$\rho_k(k)dk = \frac{2}{V} \cdot \frac{2\pi k dk}{\frac{2\pi}{L_x}\frac{2\pi}{L_y}} = \frac{2}{L_z}\frac{k}{2\pi}dk$$

$$E(k) = \frac{\hbar^2}{2m_e^*}\left[k^2 + \left(\frac{\pi}{L}\right)^2\right]$$

$$\rho_{2d}(E)dE = \rho_k(k)\frac{dk}{dE}dE = \left(\frac{2}{L_z}\frac{k}{2\pi}\right)\frac{1}{\frac{\hbar^2}{m_e^*}k}dE$$

$$\rho_{2d}(E) = \frac{m_e^*}{\pi\hbar^2 L_z}$$

2-d DOS for Multiple Energy Levels



$$\begin{cases} 0 < E < E_1 & \rho_{2d}(E) = 0 \\ E_1 < E < E_2 & \rho_{2d}(E) = \dfrac{m_e^*}{\pi\hbar^2 L_z} \\ E_2 < E < E_3 & \rho_{2d}(E) = \dfrac{2m_e^*}{\pi\hbar^2 L_z} \\ E_3 < E < E_4 & \rho_{2d}(E) = \dfrac{3m_e^*}{\pi\hbar^2 L_z} \end{cases}$$

In general

$$\rho_{2d}(E) = \frac{m_e^*}{\pi\hbar^2 L_z}\sum_{n=1}^{\infty} H(E - E_n)$$

Step Function

2-d Electron/Hole Concentration

Electron and hole concentrations:

$$n = \int_{E_C}^{\infty} f_n(E)\rho_{e,2d}(E)\,dE$$

$$p = \int_{-\infty}^{E_V} f_p(E)\rho_{h,2d}(E)\,dE$$

At T = 0K, and for $E_1 < E < E_2$

$$n = \left(F_n - E_1\right)\cdot\rho_{e,2d}(E_1 < E < E_2)$$

$$n = \left(F_n - E_1\right)\frac{m_e^*}{\pi\hbar^2 L_z}$$

Example:

10-nm-wide GaAs quantum well

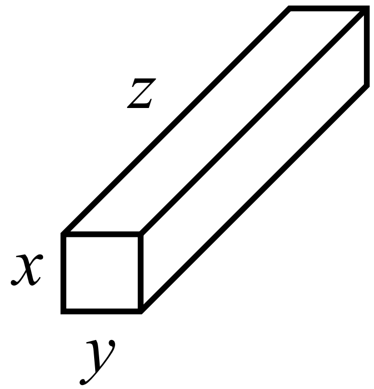
quasi-Fermi energy is 100 meV

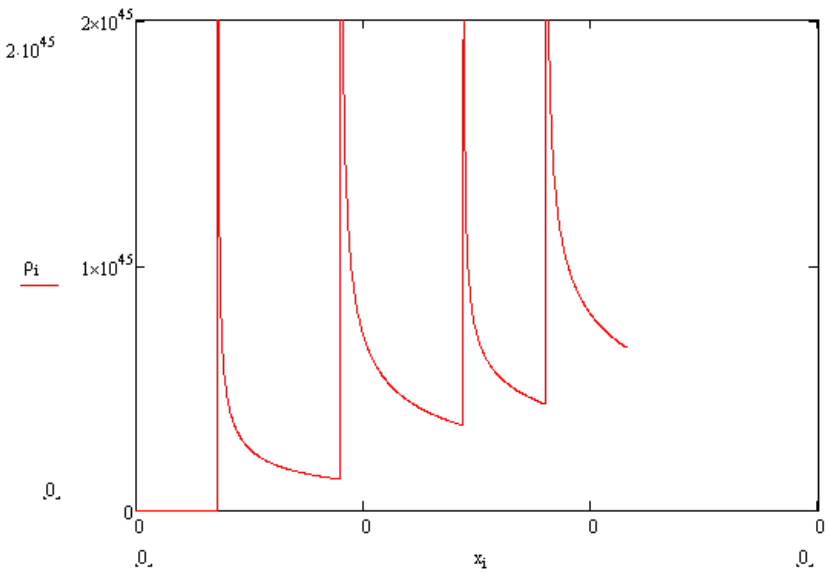
above E_1

2-d electron concentration

$m\_e := 0.067\cdot m0$	$m\_e = 6.104\times10^{-32}\,kg$
$Lz := 10nm$	$Lz = 1\times10^{-8}\,m$
$\rho 2d := \frac{m\_e}{\pi\cdot h\_bar^2\cdot Lz}$	$\rho 2d = 1.747\times10^{44}\,\frac{s^2}{kg\cdot m^5}$
$n := 100meV\cdot\rho 2d$	$n = 2.795\times10^{18}\cdot\frac{1}{cm^3}$
$n\_s := n\cdot Lz$	$n\_s = 2.795\times10^{12}\cdot\frac{1}{cm^2}$

1-d Density of States





$$E_{m,n}(k_z) = \frac{\hbar^2}{2m_e^*}\left(\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{L}\right)^2 + k_z^2\right)$$

$$dE_{m,n}(k_z) = \frac{\hbar^2}{2m_e^*} 2k_z \cdot dk_z = \frac{\hbar^2 k_z}{m_e^*} dk_z$$

$$n = \frac{2}{V}\sum_{m,n}\int_{-\infty}^{\infty}\frac{dk_z}{\left(\frac{2\pi}{L_z}\right)} = \frac{2}{\pi L_x L_y}\sum_{m,n}\int_0^{\infty} dk_z$$

$$= \frac{2}{\pi L_x L_y}\sum_{m,n}\int_0^{\infty}\frac{m_e^*}{\hbar^2 k_z} dE$$

$$= \frac{1}{\pi L_x L_y}\sqrt{\frac{2m_e^*}{\hbar^2}}\sum_{m,n}\int_0^{\infty}\frac{1}{\sqrt{E - E_{mx} - E_{ny}}} dE$$

$$\rho_{1D}(E) = \frac{1}{\pi L_x L_y}\sqrt{\frac{2m_e^*}{\hbar^2}}\sum_{m,n}\frac{1}{\sqrt{E - E_{mx} - E_{ny}}}$$